Evaluation and comparison of set-valued estimators: empirical and structural aspects

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informativeness: Statements/predictions have to be non-tautological.

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- *„mixtures“ of the first and the second approach?*

Often there is no best estimator, only undominated estimators

second approach:

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one possible (naive) answer: the estimate with the lowest area (if this exists) or the estimator with the lower expected area (independent from the true parameter) respectively.

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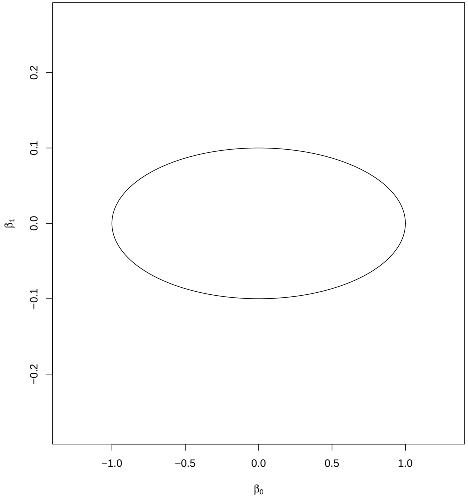

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β_1
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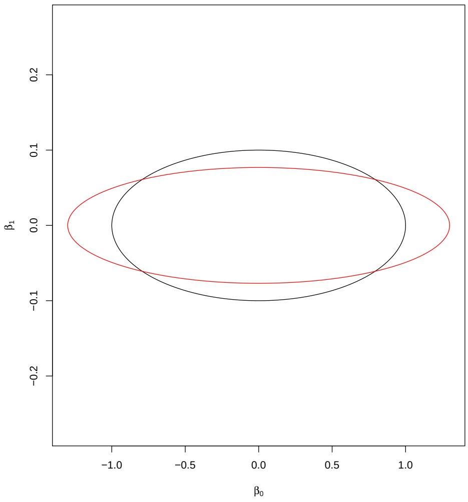
0.0
0.5
1.0
β_0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β1
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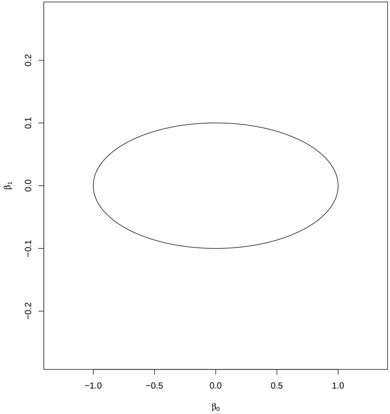
β0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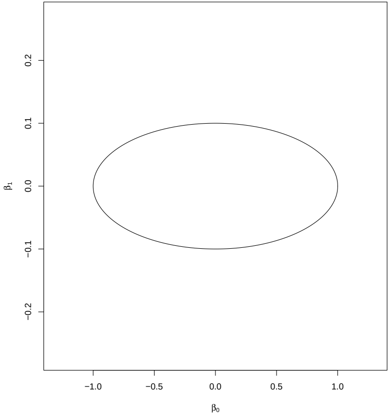
β1
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−0.1
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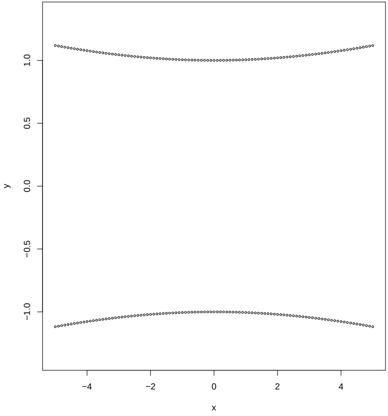
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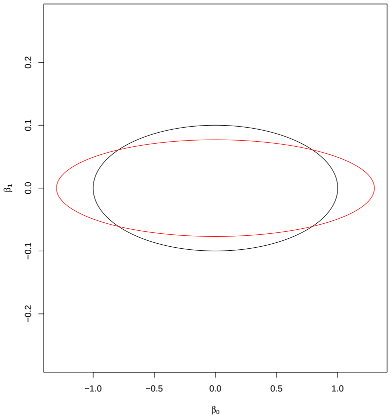
0.0
−0.5
−1.0
−4
−2
0
2
4
x


β1
0.2
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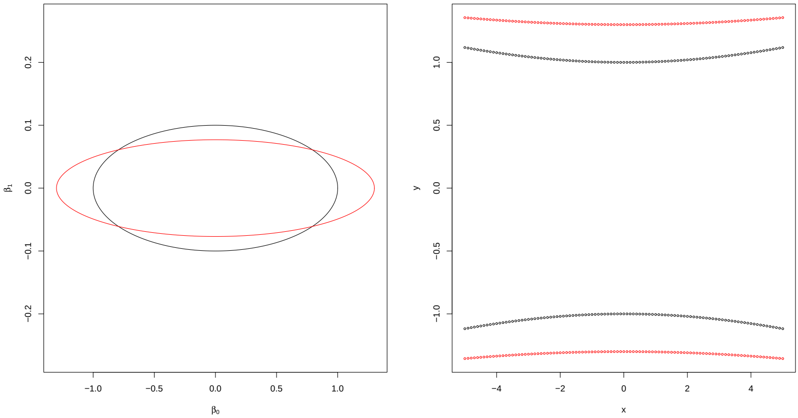

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y
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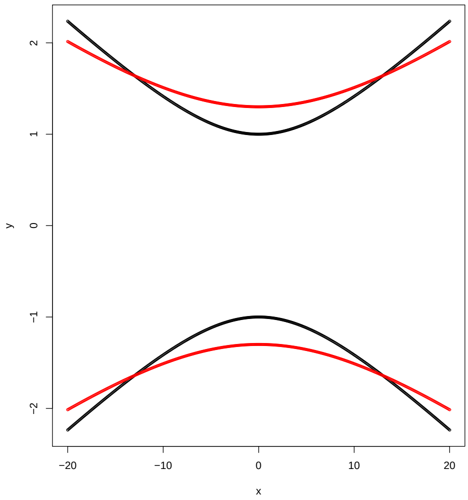
0
2
4
x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1
0
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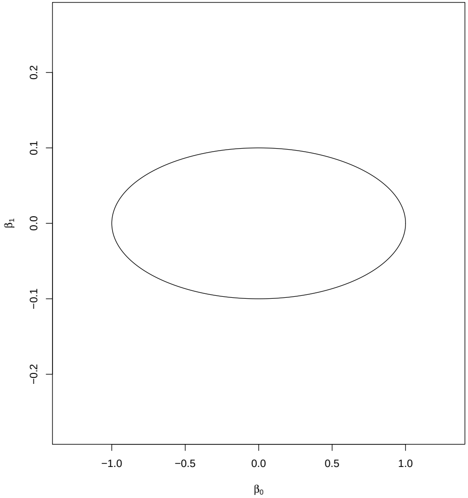
−2
−20
−10
0
10
20
x



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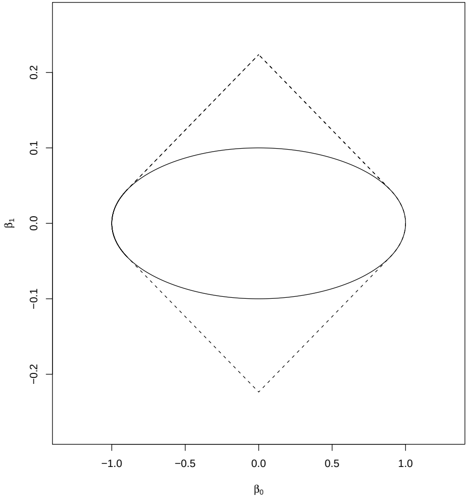

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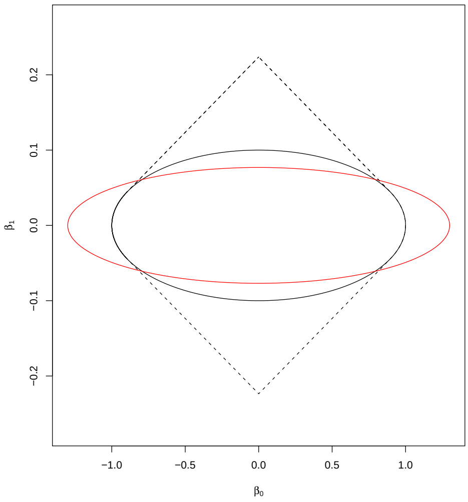
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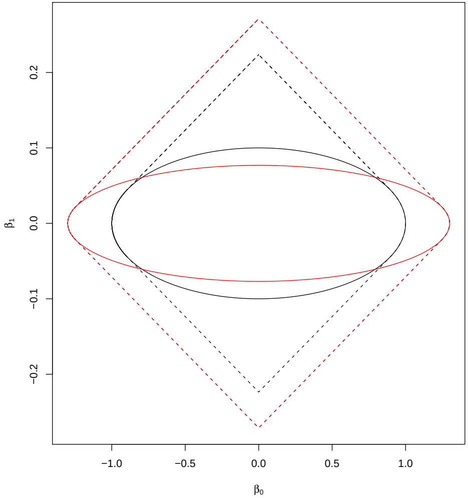
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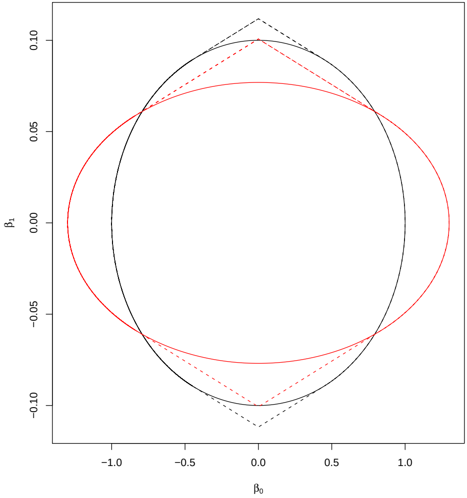
0.10
0.05
0.00
−0.05
−0.10
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0.0
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Most location parameters (e.g. median, linearly weighted mean, winsorized mean, Hodges-Lehmann-estimator) also meet the properties a') till d).

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- *it is easier to carry out simulation studies*

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  Beispiel?

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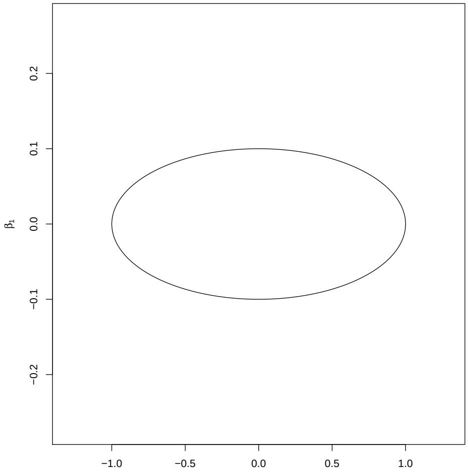

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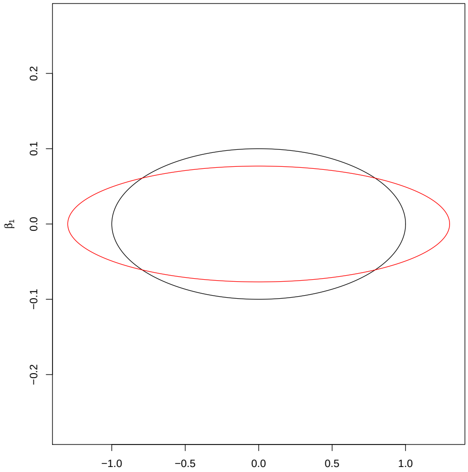

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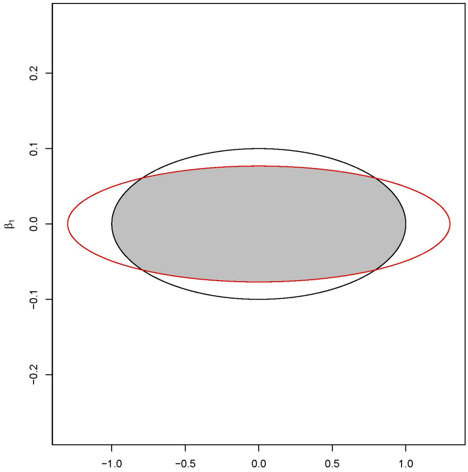

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Let $e : 2^D \longrightarrow 2^P$ be a compositional estimator.If e is undominated on the set of all one-point-sets $\binom{D}{1} = \{\{x\} | x \in D\}$with respect to a class C of <u>set-monotone</u> estimators,

Definition

let C be a class of estimators (e.g. all estimators $e : 2^D \longrightarrow 2^P$ with a coveringprobability not lower than $1-\alpha$).
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Let $e : 2^D \longrightarrow 2^P$ be a compositional estimator.If e is undominated on the set of all one-point-sets $\binom{D}{1} = \{\{x\}|x \in D\}$with respect to a class C of set-monotone estimators, then e is also undominated on 2^D.

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$\Longrightarrow$ e could not be undominated on $\binom{D}{1}$. □

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this allows definition of fuzzy-estimator in terms of the α-cut representation:
$e'(A) : [0,1] \longrightarrow 2^P : \alpha \mapsto e(A_\alpha)$.

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So if the estimator is not set-monotone, the constructed fuzzy set $e'(X)$ is in general no well defined fuzzy set.