On using different error measures for fuzzy linear regression analysis

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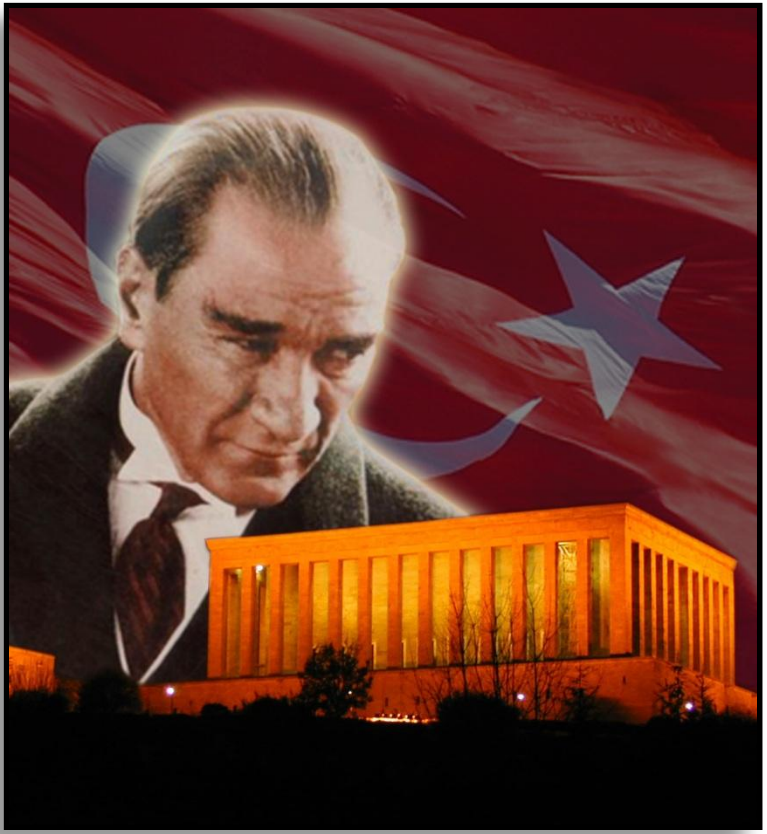


Presentation Plan

- Introduction
- Some Definitions
- Fuzzy linear regression with Monte Carlo method
- The simulation study
- Application
- Conclusion

Introduction

- Regression analysis is a statistical tool used to figure out the mathematical relation between two or more quantitative variables.
- There are many types of regression techniques in the literature. Most of these approaches are rather restrictive, and their application to real life problems requires various assumptions. Therefore new techniques have been proposed to relax some of these assumptions.
- All of these authors try to find analytical solutions for the estimators of regression parameters.
- Buckley and Abdalla [1, 2, 3] are the first practitioners of MC method into fuzzy linear regression analysis

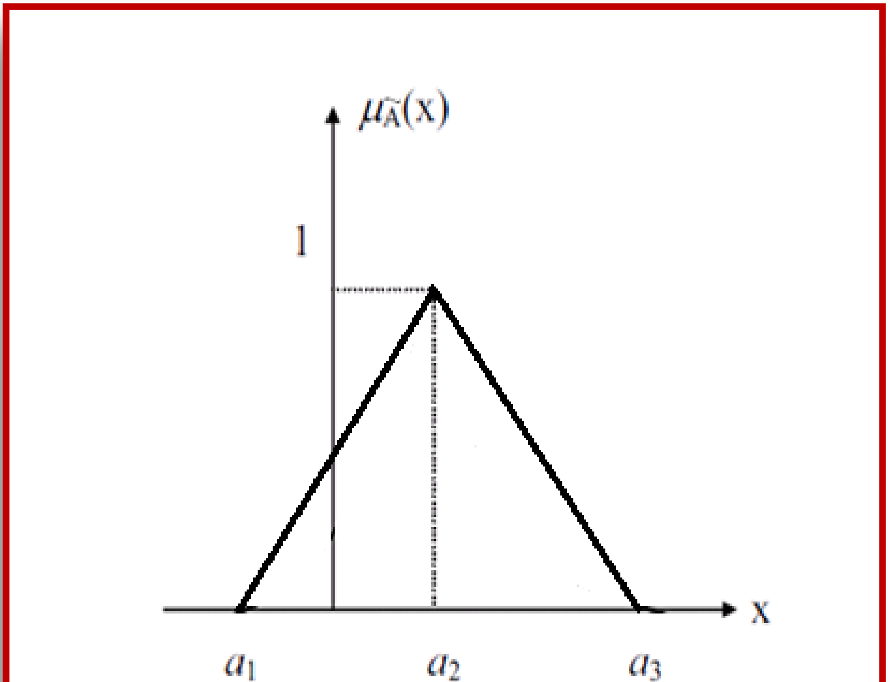
Some Definitions

A fuzzy number $\bar{A}$ is a fuzzy subset of the real line $\Re$. Its membership function $\mu_A(x)$ satisfies the following criteria :

- α-cut set of $\mu_A(x)$ is a closed interval,
- $\exists$x such that $\mu_A(x)$=1, and
- convexity such that $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_A(x_1), \mu_A(x_2))$ for $\lambda \in [0,1]$,

where, α-cut set contains all x elements that have a membership grade $\mu_A(x) \geq \alpha$.

Some Definitions



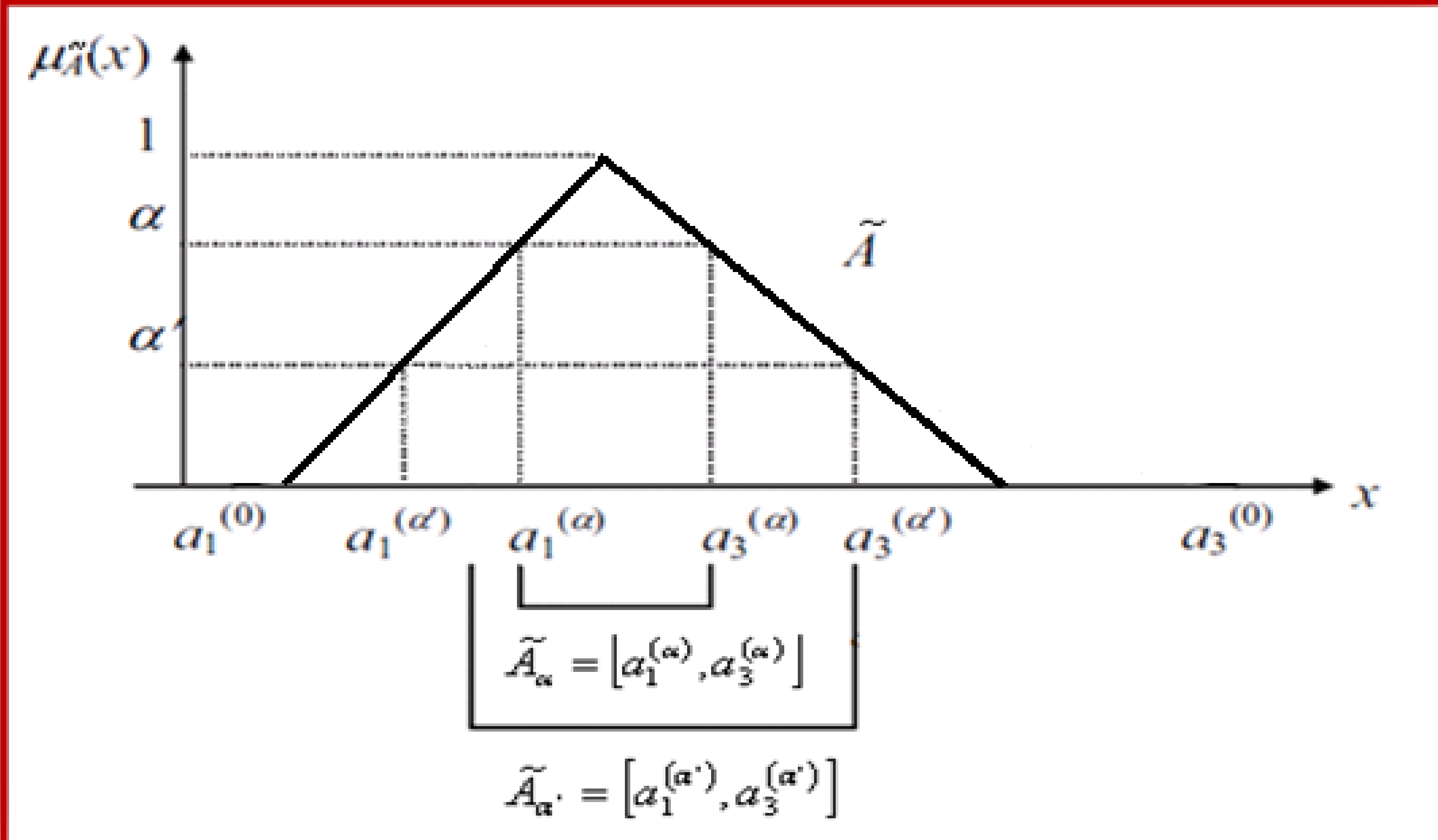
Fuzzy Number $\tilde{A} = [a_1, a_2, a_3]$

$$a_1^{\alpha} = (a_2 - a_1)\alpha + a_1$$

$$a_3^{\alpha} = -(a_3 - a_2)\alpha + a_3$$

$$[a_1^{\alpha} ; a_3^{\alpha}]$$

Some Definitions



Some Definitions

Interval Arithmetic	
$(+)$	$[a_1, a_3](+)[b_1, b_3] = [a_1 + b_1, a_3 + b_3]$
$(-)$	$[a_1, a_3](-)[b_1, b_3] = [a_1 - b_3, a_3 - b_1]$
$(\bullet)$	$[a_1, a_3](\bullet)[b_1, b_3] = [min(a_1 \bullet b_1, a_1 \bullet b_3, a_3 \bullet b_1, a_3 \bullet b_3), max(a_1 \bullet b_1, a_1 \bullet b_3, a_3 \bullet b_1, a_3 \bullet b_3)]$
$(/)$	$[a_1, a_3] / [b_1, b_3] = [min(a_1 / b_1, a_1 / b_3, a_3 / b_1, a_3 / b_3), max(a_1 / b_1, a_1 / b_3, a_3 / b_1, a_3 / b_3)]$

Some Definitions

** The absolute value of a fuzzy number $\bar{A} \in \Re_F$ is a function $F : \Re_F \to \Re_F$ denoted by $F(\bar{A}) := |\bar{A}|$ with α-cut $\bar{A}(\alpha)$. From the interval analysis [5], it is known that if $I = [I^-, I^+]$, then $|I| = [max(I^-, -I^+, 0), max(-I^-, I^+)]$, thus the α-cut of $|\bar{A}|$ is given by

$$(|\bar{A}|)_\alpha = [max(\bar{A}^-(\alpha), -\bar{A}^+(\alpha), 0), max(-\bar{A}^-(\alpha), \bar{A}^+(\alpha))]$$

and hence the absolute value of a triangular fuzzy number is given as follows

$$(|\bar{A}|)_\alpha = \begin{cases} \bar{A}(\alpha) & \text{if } \bar{A} \geq 0 \\ -\bar{A}(\alpha) & \text{if } \bar{A} \leq 0 \\ \{0, \max(-\bar{A}^-(\alpha), \bar{A}^+(\alpha))\} & \text{if } x \in (\bar{A}^-(0), \bar{A}^+(0)) \end{cases}$$

** Omar A. AbuAarqob, Nabil T. Shawagfeh and Omar A. AbuGhneim, (2008) ***Functions Defined on Fuzzy Real Numbers According to Zadeh's Extension***, International Mathematical Forum, 3,, no. 16, 763 - 776

Some Definitions

Random crisp vectors are defined as $\boldsymbol{v}_k = (v_{0k}, \ldots, v_{mk})$ where the v_{ik} are all real numbers in intervals $I_i, i = 0, 1, \ldots, m$. To obtain $\boldsymbol{v}_k$, firstly randomly crisp vectors $v_k = (x_{1k}, x_{2k}, \ldots, x_{mk})$ with all x_{ik} in $[0, 1], k = 1, 2, \ldots, N$ are needed to be generated. Since all x_{ik} starts out in [0,1], it is possible to put them into $I_i = [c_i, d_i]$ by $v_{ik} = c_i + (d_i - c_i)x_{ik}$, $i = 0, 1, \ldots, m$.

Random fuzzy vectors are defined as $\overline{\boldsymbol{V}}_k = (\overline{V}_{0k}, \ldots, \overline{V}_{mk})$, $k = 1, 2, \ldots N$, where $\overline{V}_{ik}$ are all triangular fuzzy numbers. Firstly crisp vectors $v_k = (x_{1k}, \ldots, x_{3m+3,k})$ with all the x_{ik} in $[0, 1]$, $k = 1, \ldots, N$ need to be generated. Then first three numbers in v_k are chosen and ordered from smallest to largest. If it is assumed that $x_{3k} < x_{1k} < x_{2k}$, the first triangular fuzzy number is $\overline{V}_{0k} = (x_{3k}/x_{1k}/x_{2k})$. It is possible to continue with the next three numbers in v_k, etc. making $\overline{V}_{ik}$, $i = 1, 2, \ldots, m$. In order to obtain $\overline{V}_{ik}$ be in certain intervals, it is supposed to be in interval $I_i = [a_i, b_i]$, $i = 0, 1, 2, \ldots, m$. Since each $\overline{V}_{ik}$ starts out in $[0, 1]$ it is possible to put into $[a_i, b_i]$ by computing $a_i + (b_i - a_i)x_{ik}$, $i = 1, 2, \ldots, m$.

Fuzzy linear regression with Monte Carlo method

Fuzzy regression model is classified according to the type of independent and dependent variables into three cases by Choi and Buckley [8] as the following:

(I.) Input and output data are both crisp

(II.) Input data is crisp and output data is fuzzy

(III.) Input and output data are both fuzzy

Case-II $$\overline{Y}_l = \overline{A}_0 + \overline{A}_1 x_{1l} + \overline{A}_2 x_{2l} + \ldots + \overline{A}_m x_{ml}$$

Case-III $$\overline{Y}_l = a_0 + a_1 \overline{X}_{1l} + a_2 \overline{X}_{2l} + \ldots + a_m \overline{X}_{ml}$$

Fuzzy linear regression with Monte Carlo method

Case-II $\overline{Y}_l = \overline{A}_0 + \overline{A}_1 x_{1l} + \overline{A}_2 x_{2l} + \ldots + \overline{A}_m x_{ml}$

$$\overline{Y}^*_{lk} = \overline{V}_{0k} + \overline{V}_{1k}\, x_{1l} + \ldots + \overline{V}_{mk} x_{ml}$$

Case-III $\overline{Y}_l = a_0 + a_1 \overline{X}_{1l} + a_2 \overline{X}_{2l} + \ldots + a_m \overline{X}_{ml}$

$$\overline{Y}^*_{lk} = v_{0k} + v_{1k} \overline{X}_{1l} + \ldots , v_{mk} \overline{X}_{ml}$$

$$\tilde{X}_{il} = (x_{il1} \,/\, x_{il2} \,/ x_{il3}) \qquad \tilde{Y}_l = (y_{l1} \,/\, y_{l2}/y_{l3})$$

$$\tilde{Y}_l = (y_{l1} \,/\, y_{l2}/y_{l3}) \qquad \tilde{Y}^*_{lk} = (y_{lk1} \,/\, y_{lk2}/y_{lk3})$$

Fuzzy linear regression with Monte Carlo method

$$E_{1k}(E_2) = \sum_{l=1}^{n} \left\{ \left[\int_{-\infty}^{+\infty} \boxed{|\tilde{Y}_l(x) - \tilde{Y}^*_{lk}(x)|} d_x \right] / \left[\int_{-\infty}^{+\infty} \tilde{Y}_l(x) d_x \right] \right\}$$

$$\tilde{Y}_l = [a / b / c]$$

$$\tilde{Y}_l^{\alpha} = [(b-a)\alpha + a \, ; \, -(c-b)\alpha + c]$$

$$\tilde{Y}^*_{lk} = [YY(1), YY(2), YY(3)]$$

$$\tilde{Y}^{*\alpha}_{lk} = \{[YY(2) - YY(1)]\alpha + YY(1); \, -[YY(3) - YY(2)]\alpha + YY(3)\}$$

$$\tilde{Y}_l - \tilde{Y}^*_{lk} = \{(b-a)\alpha + a - [-[YY(3) - YY(2)]\alpha + YY(3)] \, ; \, -(c-b)\alpha + c - [[YY(2) - YY(1)]\alpha + YY(1)]\}$$

Fuzzy linear regression with Monte Carlo method

$$MSE_e = \frac{1}{n}\sum_{i=1}^{n}\left[(y_{l1}-y_{lk1})^2+(y_{l2}-y_{lk2})^2+(y_{l3}-y_{lk3})^2\right]$$

$$MPE_e = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{y_{lk1}-y_{l1}}{y_{l1}}+\frac{y_{lk2}-y_{l2}}{y_{l2}}+\frac{y_{lk3}-y_{l3}}{y_{l3}}\right]$$

$$MAPE_e = \frac{100}{n}\sum_{i=1}^{n}\left[\left|\frac{y_{lk1}-y_{l1}}{y_{l1}}\right|+\left|\frac{y_{lk2}-y_{l2}}{y_{l2}}\right|+\left|\frac{y_{lk3}-y_{l3}}{y_{l3}}\right|\right]$$

$$SMAPE_e = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{|y_{lk1}-y_{l1}|}{(y_{lk1}-y_{l1})/2}+\frac{|y_{lk2}-y_{l2}|}{(y_{lk2}-y_{l2})/2}+\frac{|y_{lk3}-y_{l3}|}{(y_{lk3}-y_{l3})/2}\right]$$

The simulation study

Intervals for Case-II and Case-III.

	I_0	I_1	I_2	I_3	I_4	I_5
$\overline{A}_0$ (or a_0)	[0,3]	[-2,1]	[2,15]	[-12,15]	[-3,-2]	[-22,-4.2]
$\overline{A}_1$ (or a_1)	[0,2]	[-1,1]	[10,22]	[-3,27]	[-1.756,0]	[-28,-3.5]
$\overline{A}_2$ (or a_2)	[3,4.5]	[-2.5,1.5]	[4,30]	[-45,18]	[-4.8,-3.75	[-18,-1]
$\overline{A}_3$	[1.2,2.4]	[-1.2,1.4]	[17,35]	[-24,28]	[-1.02,0]	[-27,-7]

$$MSE_c = \frac{1}{3}\sum_{j=1}^{3}(y_{lj} - y_{lkj})^2$$

"comparison measure"

$$MAE_c = \frac{1}{3}\sum_{j=1}^{3}\left|y_{lj} - y_{lkj}\right|$$

The simulation study

Simulation results of Case-II for MAE_c.

Error	Coef.	I_0	I_1	I_2
E_1	$\overline{A}_0$	0.851 0.934 1.036	1.348 0.946 0.776	4.286 5.689 8.359
	$\overline{A}_1$	1.039 0.818 0.631	1.332 1.229 1.138	9.278 10.018 11.103
	$\overline{A}_2$	4.120 4.161 4.278	0.634 0.499 0.536	7.059 7.908 10.618
	$\overline{A}_3$	3.369 3.393 3.482	0.926 0.924 1.073	19.681 20.285 22.491
		I_3	I_4	I_5
	$\overline{A}_0$	5.756 5.756 6.038	3.386 3.301 3.105	15.581 8.988 7.693
	$\overline{A}_1$	2.131 1.671 4.034	2.336 2.181 2.094	10.873 8.669 6.406
	$\overline{A}_2$	2.934 3.140 4.187	2.951 2.864 2.829	8.210 4.038 1.898
	$\overline{A}_3$	4.084 1.419 3.713	1.154 1.342 1.443	10.417 7.689 5.987
E_2		I_0	I_1	I_2
	$\overline{A}_0$	0.851 0.934 1.036	1.348 0.946 0.776	4.286 5.689 8.359
	$\overline{A}_1$	1.039 0.818 0.631	1.332 1.229 1.138	9.278 10.018 11.103
	$\overline{A}_2$	4.120 4.161 4.278	0.634 0.499 0.536	7.059 7.908 10.618
	$\overline{A}_3$	3.369 3.393 3.482	0.926 0.924 1.073	19.681 20.284 22.491
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	$\overline{A}_2$	2.934 3.140 4.187	2.951 2.864 2.829	8.210 4.038 1.898
	$\overline{A}_3$	4.084 1.419 3.713	1.154 1.342 1.443	10.417 7.689 5.987
MSE_e		I_0	I_1	I_2
	$\overline{A}_0$	0.803 0.898 1.042	1.306 0.939 0.779	4.050 5.171 7.726
	$\overline{A}_1$	1.043 0.833 0.590	1.321 1.233 1.142	9.264 9.862 10.792
	$\overline{A}_2$	4.118 4.154 4.262	0.619 0.485 0.473	6.773 7.508 10.162
	$\overline{A}_3$	3.364 3.385 3.454	0.911 0.900 1.042	19.669 20.267 22.528
		I_3	I_4	I_5
	$\overline{A}_0$	5.353 6.379 7.052	3.359 3.296 3.086	14.871 9.322 7.928
	$\overline{A}_1$	2.741 2.217 3.063	2.300 2.142 2.070	10.740 9.234 6.755
	$\overline{A}_2$	3.030 3.549 4.046	2.929 2.851 2.824	7.322 3.856 1.964
	$\overline{A}_3$	3.070 1.311 3.828	1.138 1.352 1.459	10.062 7.531 6.122

The simulation study

Simulation results of Case-II for MAE_c (Cont.).

Error	Coef.	I_0	I_1	I_2
MPE_e	$\overline{A}_0$	0.922 1.089 1.160	2.146 1.556 1.032	4.465 7.279 9.744
	$\overline{A}_1$	1.648 1.379 1.021	2.635 2.327 1.903	10.124 12.078 14.680
	$\overline{A}_2$	4.631 4.853 5.044	1.491 1.571 1.702	16.537 20.074 23.419
	$\overline{A}_3$	3.853 4.035 4.120	2.217 2.599 2.848	28.592 31.570 33.516
		I_3	I_4	I_5
	$\overline{A}_0$	7.744 7.270 8.515	3.708 3.517 3.360	18.090 14.424 11.295
	$\overline{A}_1$	5.247 7.617 12.219	3.449 3.238 2.949	25.943 22.565 18.028
	$\overline{A}_2$	25.307 21.951 19.343	3.346 3.197 3.078	10.216 7.264 4.817
	$\overline{A}_3$	19.449 21.508 23.517	1.572 1.700 1.743	14.469 10.900 8.524
$MAPE_e$		I_0	I_1	I_2
	$\overline{A}_0$	0.788 0.856 0.981	1.308 0.961 0.648	4.428 5.870 8.551
	$\overline{A}_1$	1.561 1.350 1.145	1.623 1.361 1.223	9.063 9.800 11.240
	$\overline{A}_2$	4.106 4.138 4.264	0.498 0.592 0.817	8.957 10.375 13.032
	$\overline{A}_3$	3.484 3.523 3.629	1.143 1.231 1.393	19.808 20.655 23.022
		I_3	I_4	I_5
	$\overline{A}_0$	5.902 5.815 6.503	3.454 3.333 3.202	15.270 10.213 8.363
	$\overline{A}_1$	2.646 2.579 4.541	2.582 2.406 2.228	11.503 8.578 6.475
	$\overline{A}_2$	4.493 3.783 4.359	2.952 2.857 2.821	8.870 4.749 2.360
	$\overline{A}_3$	5.018 2.389 4.174	1.226 1.404 1.516	11.336 7.915 6.310
$SMAPE_e$		I_0	I_1	I_2
	$\overline{A}_0$	0.913 1.058 1.339	1.419 0.939 0.631	5.469 7.663 9.895
	$\overline{A}_1$	0.694 0.418 0.222	1.568 1.302 1.175	12.620 14.507 16.509
	$\overline{A}_2$	4.517 4.754 5.067	0.617 0.566 0.784	14.614 17.740 21.796
	$\overline{A}_3$	3.506 3.622 3.839	1.064 1.163 1.304	25.040 28.222 31.373
		I_3	I_4	I_5
	$\overline{A}_0$	6.311 3.597 7.250	3.643 3.462 3.293	16.571 12.775 9.781
	$\overline{A}_1$	2.538 4.116 11.868	2.854 2.514 2.297	17.077 12.487 9.821
	$\overline{A}_2$	13.031 3.094 5.363	3.514 3.314 3.138	10.999 8.009 5.561
	$\overline{A}_3$	10.425 3.871 5.468	1.255 1.308 1.408	18.952 14.890 11.485

The simulation study

Simulation results of Case-II for MSE_c.

Error	Coef.	I_0	I_1	I_2
E_1	$\overline{A}_0$	0.915 1.156 1.601	2.041 1.084 0.770	28.992 46.899 93.709
	$\overline{A}_1$	1.540 1.080 0.817	1.800 1.518 1.298	87.442 102.562 128.575
	$\overline{A}_2$	16.988 17.331 18.328	0.492 0.322 0.562	54.759 72.757 129.412
	$\overline{A}_3$	11.365 11.548 12.172	0.871 0.892 1.232	387.422 412.036 507.389
		I_3	I_4	I_5
	$\overline{A}_0$	40.414 36.512 44.080	11.505 10.923 9.649	260.225 89.469 63.315
	$\overline{A}_1$	7.311 4.999 27.971	5.536 4.812 4.407	126.779 80.511 42.360
	$\overline{A}_2$	15.628 13.166 19.415	8.740 8.213 8.009	82.772 21.568 6.159
	$\overline{A}_3$	25.974 2.967 18.701	1.350 1.842 2.139	121.613 62.725 36.864
E_2		I_0	I_1	I_2
	$\overline{A}_0$	0.915 1.156 1.601	2.041 1.084 0.770	28.992 46.899 93.709
	$\overline{A}_1$	1.540 1.080 0.817	1.800 1.518 1.298	87.442 102.562 128.575
	$\overline{A}_2$	16.988 17.331 18.328	0.492 0.322 0.562	54.759 72.757 129.412
	$\overline{A}_3$	11.365 11.548 12.172	0.871 0.892 1.232	387.421 412.036 507.389
		I_3	I_4	I_5
	$\overline{A}_0$	40.414 36.512 44.080	11.505 10.923 9.649	260.225 89.469 63.315
	$\overline{A}_1$	7.311 4.999 27.971	5.536 4.812 4.407	126.779 80.511 42.360
	$\overline{A}_2$	15.628 13.166 19.415	8.740 8.213 8.009	82.772 21.568 6.159
	$\overline{A}_3$	25.974 2.967 18.701	1.350 1.842 2.139	121.613 62.725 36.864
MSE_e		I_0	I_1	I_2
	$\overline{A}_0$	0.839 1.067 1.606	1.864 1.007 0.733	26.043 39.383 83.920
	$\overline{A}_1$	1.520 1.086 0.740	1.759 1.523 1.306	86.787 98.699 120.272
	$\overline{A}_2$	16.972 17.268 18.184	0.454 0.296 0.457	48.139 62.361 114.398
	$\overline{A}_3$	11.330 11.490 11.975	0.839 0.841 1.159	386.928 411.306 508.555
		I_3	I_4	I_5
	$\overline{A}_0$	35.701 43.290 57.721	11.315 10.885 9.527	238.789 98.453 67.920
	$\overline{A}_1$	10.439 7.122 19.404	5.340 4.616 4.292	121.004 91.898 47.290
	$\overline{A}_2$	17.602 15.308 17.534	8.600 8.136 7.979	64.678 19.681 5.863
	$\overline{A}_3$	17.643 2.475 21.396	1.304 1.857 2.175	111.155 59.617 38.222

The simulation study

Simulation results of Case-II for MSE_c (Cont.).

Error	Coef.	I_0	I_1	I_2
MPE_e	$\overline{A}_0$	1.027 1.492 1.851	5.332 3.480 1.975	30.615 70.288 109.625
	$\overline{A}_1$	2.966 2.343 1.529	7.143 5.769 4.015	110.021 158.699 230.036
	$\overline{A}_2$	21.699 23.793 25.660	2.517 2.987 3.576	379.315 503.296 627.930
	$\overline{A}_3$	14.996 16.414 17.057	5.753 7.460 8.572	860.502 1031.404 1147.233
		I_3	I_4	I_5
	$\overline{A}_0$	76.561 67.400 87.248	13.826 12.474 11.377	351.420 241.841 155.800
	$\overline{A}_1$	45.128 107.093 228.295	12.106 10.833 9.101	710.537 574.032 399.856
	$\overline{A}_2$	874.030 636.206 461.368	11.326 10.343 9.577	138.653 85.045 50.251
	$\overline{A}_3$	415.632 517.890 618.216	2.601 2.993 3.105	259.960 159.906 98.891
$MAPE_e$		I_0	I_1	I_2
	$\overline{A}_0$	0.757 0.961 1.403	2.009 1.158 0.581	29.651 49.680 95.049
	$\overline{A}_1$	2.658 2.113 1.682	2.756 1.905 1.529	82.906 97.548 131.604
	$\overline{A}_2$	16.869 17.139 18.231	0.334 0.451 0.872	106.546 149.771 220.593
	$\overline{A}_3$	12.208 12.525 13.284	1.373 1.616 2.126	392.967 429.015 534.298
		I_3	I_4	I_5
	$\overline{A}_0$	43.918 39.397 51.861	12.010 11.174 10.293	252.837 123.990 80.857
	$\overline{A}_1$	11.279 13.875 40.021	6.941 6.026 5.088	145.541 79.114 43.186
	$\overline{A}_2$	36.686 19.687 25.012	8.744 8.177 7.963	100.968 35.562 14.196
	$\overline{A}_3$	43.116 12.747 29.355	1.547 2.043 2.375	147.821 69.402 41.943
$SMAPE_e$		I_0	I_1	I_2
	$\overline{A}_0$	1.044 1.479 2.175	2.422 1.157 0.557	42.615 73.014 109.601
	$\overline{A}_1$	0.775 0.359 0.147	2.604 1.727 1.396	171.104 223.411 283.245
	$\overline{A}_2$	20.600 22.808 25.839	0.534 0.444 0.892	269.422 379.326 532.499
	$\overline{A}_3$	12.370 13.236 14.881	1.181 1.438 1.831	652.037 827.301 1010.590
		I_3	I_4	I_5
	$\overline{A}_0$	52.589 20.186 67.238	13.351 12.077 10.924	297.198 188.286 113.476
	$\overline{A}_1$	10.761 28.492 178.353	8.522 6.599 5.445	346.732 199.206 124.402
	$\overline{A}_2$	286.051 17.092 52.909	12.421 11.076 9.944	143.533 87.925 50.159
	$\overline{A}_3$	136.576 24.055 66.821	1.642 1.809 2.096	386.544 251.038 156.263

The simulation study

Simulation results of Case-III for MAE_c.

Error	Coef.	I_0	I_1	I_2	I_3	I_4	I_5
$E1$	a_0	0.883	0.812	3.162	0.719	1.175	5.533
	a_1	1.098	1.252	8.144	0.501	2.019	5.657
	a_2	4.003	1.387	5.126	0.825	2.757	0.483
$E2$	a_0	0.883	0.812	3.162	0.719	1.175	5.533
	a_1	1.098	1.252	8.144	0.501	2.019	5.657
	a_2	4.003	1.387	5.126	0.825	2.757	0.483
MSE_e	a_0	0.841	0.774	2.980	0.689	1.037	5.532
	a_1	1.139	1.229	8.137	0.508	2.015	5.658
	a_2	4.002	1.420	5.136	0.872	2.756	0.451
MPE_e	a_0	1.478	1.429	7.903	13.036	2.372	13.708
	a_1	1.795	2.276	9.255	8.530	2.657	14.616
	a_2	4.516	1.601	13.837	17.634	2.973	3.509
$MAPE_e$	a_0	1.082	0.949	3.764	0.978	1.609	5.815
	a_1	1.609	1.130	8.397	0.531	2.056	5.707
	a_2	4.006	0.823	5.117	0.692	2.764	0.593
$SMAPE_e$	a_0	1.017	0.926	5.447	0.938	1.680	11.497
	a_1	0.145	1.263	9.759	0.865	2.453	9.805
	a_2	4.315	1.172	6.162	0.915	3.008	9.135

The simulation study

Simulation results of Case-III for MSE_c.

Error	Coef.	I_0	I_1	I_2	I_3	I_4	I_5
$E1$	a_0	1.002	1.005	13.280	59.308	2.292	30.557
	a_1	1.503	1.506	66.337	26.324	4.079	32.006
	a_2	16.024	16.025	26.286	292.011	7.601	0.354
$E2$	a_0	1.002	1.005	13.280	59.308	2.292	30.557
	a_1	1.503	1.506	66.337	26.324	4.079	32.006
	a_2	16.024	16.025	26.286	292.011	7.601	0.354
MSE_e	a_0	0.889	0.891	10.950	43.481	1.806	30.531
	a_1	1.513	1.517	66.221	26.414	4.060	32.014
	a_2	16.017	16.018	26.386	291.914	7.596	0.276
MPE_e	a_0	2.446	2.449	100.537	170.782	7.879	259.572
	a_1	3.530	3.530	96.317	318.457	7.729	346.371
	a_2	20.881	20.882	338.231	615.259	9.001	55.171
$MAPE_e$	a_0	1.470	1.473	26.526	98.067	4.168	35.910
	a_1	2.909	2.913	71.604	28.301	4.265	32.599
	a_2	16.052	16.052	26.332	293.124	7.641	1.978
$SMAPE_e$	a_0	1.389	1.385	45.644	81.992	4.366	170.156
	a_1	0.118	0.118	104.065	104.777	6.314	134.537
	a_2	18.865	18.865	43.149	411.546	9.203	111.423

Application

Case-II $\overline{Y}_l = \overline{A}_0 + \overline{A}_1 x_{1l} + \overline{A}_2 x_{2l} + \ldots + \overline{A}_m x_{ml}$

Fuzzy Output	x_1	x_2	x_3
(2.27/5.83/9.39)	2.00	0.00	15.25
(0.33/0.85/1.37)	0.00	5.00	14.13
(5.43/13.93/22.43)	1.13	1.50	14.13
(1.56/4.00/6.44)	2.00	1.25	13.63
(0.64/1.65/2.66)	2.19	3.75	14.75
(0.62/1.58/2.54)	0.25	3.50	13.75
(3.19/8.18/13.17)	0.75	5.25	15.25
(0.72/1.85/2.98)	4.25	2.00	13.50

The intervals for I_i, $i = 0, 1, 2, 3$ for Case-II.

	MCI	$MCII$	$MCIII$	$MCIV$
I_0	[-1,0]	[0,1]	[-18.174,-18.174]	[28.000,47.916]
I_1	[-1,0]	[-1,0]	[-1.083,-1.083]	[-2.542,-2.542]
I_2	[-1.5,-0.5]	[-1.5,-0.5]	[-1.500,-1.500]	[-2.333,-2.333]
I_3	[0,1]	[0,1]	[1.733,2.149]	[-1.354,-1.354]

Application

Case-II $\overline{Y}_l = \overline{A}_0 + \overline{A}_1 x_{1l} + \overline{A}_2 x_{2l} + \ldots + \overline{A}_m x_{ml}$

Estimates of coefficients under *MCI-MCII-MCIII-MCIV* setting for Case-II.

		$\overline{A}_0$	$\overline{A}_1$	$\overline{A}_2$	$\overline{A}_3$
MCI	E_1	-0.654 -0.163 -0.139	-0.285 -0.228 -0.133	-0.643 -0.555 -0.543	0.304 0.317 0.321
	E_2	-0.754 -0.548 -0.421	-0.802 -0.786 -0.684	-1.323 -1.265 -1.251	0.548 0.566 0.661
	MSE_e	-0.712 -0.672 -0.611	-0.934 -0.928 -0.887	-1.202 -1.139 -1.035	0.613 0.758 0.801
	MPE_e	-0.938 -0.836 -0.251	-0.950 -0.810 -0.492	-1.426 -1.349 -1.016	0.010 0.042 0.066
$MCII$	E_1	0.061 0.316 0.341	-0.271 -0.268 -0.129	-0.822 -0.727 -0.721	0.259 0.294 0.336
	E_2	0.767 0.901 0.923	-0.604 -0.430 -0.145	-1.096 -1.083 -1.015	0.355 0.367 0.517
	MSE_e	0.210 0.262 0.937	-0.970 -0.882 -0.771	-1.285 -1.245 -0.998	0.530 0.629 0.686
	MPE_e	0.062 0.164 0.749	-0.950 -0.891 -0.492	-1.426 -1.349 -1.016	0.010 0.042 0.066
$MCIII$	E_1	-18.174 -18.174 -18.174	-1.083 -1.083 -1.083	-1.500 -1.500 -1.500	1.875 1.876 1.879
	E_2	-18.174 -18.174 -18.174	-1.083 -1.083 -1.083	-1.500 -1.500 -1.500	1.823 1.888 1.960
	MSE_e	-18.174 -18.174 -18.174	-1.083 -1.083 -1.083	-1.500 -1.500 -1.500	1.904 2.015 2.119
	MPE_e	-18.174 -18.174 -18.174	-1.083 -1.083 -1.083	-1.500 -1.500 -1.500	1.736 1.739 1.741
$MCIV$	E_1	30.645 30.645 30.658	-2.542 -2.542 -2.542	-2.333 -2.333 -2.333	-1.354 -1.354 -1.354
	E_2	31.102 35.335 36.042	-2.542 -2.542 -2.542	-2.333 -2.333 -2.333	-1.354 -1.354 -1.354
	MSE_e	31.013 35.597 36.814	-2.542 -2.542 -2.542	-2.333 -2.333 -2.333	-1.354 -1.354 -1.354
	MPE_e	28.168 28.168 28.664	-2.542 -2.542 -2.542	-2.333 -2.333 -2.333	-1.354 -1.354 -1.354

$\overline{A}_0 = (-0.710/-0.539/-0.524)$ $\overline{A}_2 = (-1.090/-1.089/-1.088)$

$\overline{A}_1 = (-0.610/-0.473/-0.472)$ $\overline{A}_3 = (0.459/0.487/0.68)$

Application

Case-II $\overline{Y}_l = \overline{A}_0 + \overline{A}_1 x_{1l} + \overline{A}_2 x_{2l} + \ldots + \overline{A}_m x_{ml}$

Comparison of error measures in the application (Case-II).

				MCI		$MCII$		$MCIII$		$MCIV$	
Error	[18]	[20]	[14]	[3]	MC	[3]	MC	[3]	MC	[3]	MC
E_1	53.82	48.79	16.98	6.17	9.00	5.81	9.49	7.13	6.83	8.20	7.34
E_2	143.45	131.83	70.99	64.89	63.26	63.59	64.06	66.46	66.42	94.09	94.26
MSE_e	NA	NA	NA	NA	27.18	NA	1.78	NA	26.23	NA	41.03
MPE_e	NA	NA	NA	NA	-3.30	NA	-2.88	NA	-0.81	NA	-1.70

Application

Case-III $\overline{Y}_l = a_0 + a_1 \overline{X}_{1l} + a_2 \overline{X}_{2l} + \ldots + a_m \overline{X}_{ml}$

Fuzzy Output	$\bar{X}_1$	$\bar{X}_2$
(55.4/61.6/64.7)	(5.7/6.0/6.9)	(5.4/6.3/7.1)
(50.5/53.2/58.5)	(4.0/4.4/5.1)	(4.7/5.5/5.8)
(55.7/65.5/75.3)	(8.6/9.1/9.8)	(3.4/3.6/4.0)
(61.7/64.9/74.7)	(6.9/8.1/9.3)	(5.0/5.8/6.7)
(69.1/72.7/80.0)	(8.7/9.4/11.2)	(6.5/6.8/7.1)
(49.6/52.2/57.4)	(4.6/4.8/5.5)	(6.7/7.9/8.7)
(47.7/50.2/55.2)	(7.2/7.6/8.7)	(4.0/4.2/4.8)
(41.8/44.0/48.4)	(4.2/4.4/4.8)	(5.4/6.0/6.3)
(45.7/53.8/61.9)	(8.2/9.1/10.0)	(2.7/2.8/3.2)
(45.4/53.5/58.9)	(6.0/6.7/7.4)	(5.7/6.7/7.7)

The intervals for I_i, $i = 0, 1, 2$ for Case-III.

	MCI	$MCII$	$MCIII$	$MCIV$
I_0	[0,5]	[0,37]	[16.528,16.528]	[33.808,36.601]
I_1	[0,6]	[0,6]	[3.558,3.982]	[1.294,3.765]
I_2	[0,4]	[0,6]	[2.575,2.575]	[0.473,0.473]

The simulation study

Estimates of coefficients under *MCI-MCII-MCIII-MCIV* setting for Case-III.

		a_0	a_1	a_2
MCI	E_1	2.657	0.013	0.006
	E_2	4.849	4.882	3.198
	MSE_e	4.919	4.642	3.544
	MPE_e	0.379	0.027	0.055
$MCII$	E_1	19.661	0.013	0.010
	E_2	9.970	4.458	2.850
	MSE_e	14.540	4.009	2.699
	MPE_e	0.312	0.051	0.200
$MCIII$	E_1	16.528	3.558	2.575
	E_2	16.528	3.807	2.575
	MSE_e	16.528	3.809	2.575
	MPE_e	16.528	3.558	2.575
$MCIV$	E_1	36.519	1.295	0.473
	E_2	33.822	3.294	0.473
	MSE_e	33.810	3.053	0.473
	MPE_e	33.835	1.296	0.473

Comparison of error measures in the application (Case-III).

				MCI		$MCII$		$MCIII$		$MCIV$	
Error	[10]	[8]	[13]	[1]	MC	[1]	MC	[1]	MC	[1]	MC
E_1	13.58	11.11	12.03	10.02	10.03	9.39	10.03	12.73	15.90	9.59	11.75
E_2	141.63	137.85	NA	133.11	130.16	133.12	129.71	146.53	137.83	170.12	161.07
MSE_e	NA	NA	NA	NA	76.72	NA	72.15	NA	72.72	NA	98.68
MPE_e	NA	NA	NA	NA	-0.99	NA	-0.97	NA	-0.02	NA	-0.20

Conclusion

- In this study, we use different error measures to find the parameter estimates of fuzzy linear regression models with MC method.

- A simulation study is conducted to compare the estimation performances of the error measures we mentioned. We showed that only two error measures (E1 and E2) are not enough for estimating the parameters of fuzzy linear regression models.

- We also estimate the parameters with considering five different intervals from where they come.

- it is possible to say that best error measures to estimate fuzzy/crisp parameters of fuzzy regression models are not only E1 and E2 but also MSEe. Furthermore the worst error measure is MPEe for estimating the parameters of fuzzy regression models.

Future Works

- Considering more than one way to get the absolute value of the triangular fuzzy number, it is possible to apply different methods in MC method in fuzzy linear regression analysis.

- Extension of the proposed method for different type of fuzzy regression models, such as nonparametric fuzzy regression or fuzzy nonlinear regression, is a potential area for the future work.

- The most important thing for fuzzy linear regression model is deciding the intervals about the parameters. New methods can be applied to choose the convenient intervals. For example expert systems or fuzzy expert systems.

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On using different error measures for fuzzy linear regression analysis

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