On the Robustness of Imprecise Probability Methods

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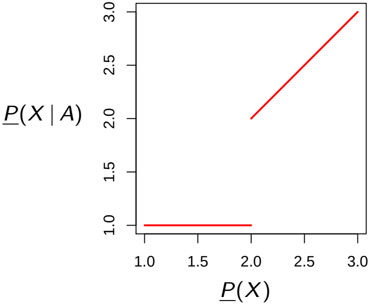
introduction

- IP methods are often claimed to be robust (or more robust than conventional methods): **are they really robust?**
- "robustness signifies insensitivity to small deviations from the assumptions" (Huber, 1981, p. 1)
- in the IP approach:
  - probability values $P(A)$ need not be precisely chosen, they are replaced by intervals $[\underline{P}(A), \overline{P}(A)]$,
  - but this means choosing two values precisely: $\underline{P}(A)$ and $\overline{P}(A)$
- the robustness of the conventional methods refers to the arbitrariness in the choice of $P(A)$, while the robustness of the IP methods refers to the arbitrariness in the choices of $\underline{P}(A)$ and $\overline{P}(A)$

robust or not robust?

- natural extension of IP models: **robust** (Troffaes and Hable, 2011)
- updating of IP models (by natural/regular extension): **not robust** in general
- e.g., $X \in \{1, 2, 3\}$, unique assessment: $\underline{P}(X)$, observation: $A = \{X \neq 2\}$



- by contrast, updating of precise probabilities is continuous

doubtful assumptions

- "conclusions drawn from the imprecise model are automatically robust, because they do not rely on arbitrary or doubtful assumptions"
  (Walley, 1991, p. 5)

- e.g., sequence of binary experiments, starting with "complete ignorance":
  - Bayesian approach:
    - $X_1, X_2, \ldots \overset{i.i.d.}{\sim} Ber(\theta)$ conditional on θ (exchangeability)
    - $\theta \sim Beta(s, t)$ (conjugate prior)
    - $t = 1/2$
    - $s = ?$
  - IP approach:
    - $X_1, X_2, \ldots \overset{i.i.d.}{\sim} Ber(\theta)$ conditional on θ (exchangeability)
    - $\theta \sim Beta(s, t)$ (conjugate priors)
    - $t \in (0, 1)$
    - $s = ?$

misleading comparisons

- imprecise methods based on IP models are often compared with precise methods based on precise probabilities
- e.g., imprecise classifiers based on IDM priors are compared with Bayesian classifiers based on uniform priors
- in some situations, imprecise methods can be more robust, but they can be based on precise probabilities as well
- the gain in robustness is obtained by allowing the methods to be inconclusive, and not necessarily by basing them on IP models

conclusion

- there seems to be no reason to claim that IP methods are in general robust (or more robust than conventional methods)
- this is particularly important in statistics, where the (higher) robustness of the IP approach could have been one of the few general advantages over the Bayesian approach
- of course, IP models can be used to study the robustness of Bayesian methods

references

- Huber, P. J. (1981). *Robust Statistics*. Wiley.
- Troffaes, M. C. M., and Hable, R. (2011). Robustness of natural extension. In *ISIPTA '11*. SIPTA, 361–370.
- Walley, P. (1991). *Statistical Reasoning with Imprecise Probabilities*. Chapman and Hall.