Three contrasts between two senses of *coherence*

Teddy Seidenfeld

***Joint work with M.J.Schervish and J.B.Kadane* – Statistics, CMU**

Call an agent's choices *coherent* when they respect *simple dominance* relative to a (finite) partition.

$\Omega = \{\omega_1, \ldots, \omega_n\}$ is a finite partition of the sure event: a set of *states*.

Consider two acts A_1, A_2 defined by the their outcomes relative to Ω.

	ω_1	ω_2	ω_3	…	ω_n
A_1	o_{11}	o_{12}	o_{13}	…	o_{1n}
A_2	o_{21}	o_{22}	o_{23}	…	o_{2n}

Suppose the agent can compare the desirability of different outcomes at least within each state, and, for each state ω_j, outcome o_{2j} is (strictly) preferred to outcome o_{1j}, $j = 1, \ldots, n$. Then A_2 *simply dominates* A_1 with respect to Ω.

- ***Coherence*: When A_2 simply dominates A_1 in some finite partition, then A_1 is inadmissible in any choice problem where A_2 is feasible.**

<u>Background on de Finetti's two senses of *coherence*</u>

De Finetti (1937, 1974) developed two senses of *coherence* (*coherence*$_1$ and *coherence*$_2$), which he extended also to infinite partitions.

Let $\Omega = \{\omega_1, \ldots, \omega_n, \ldots\}$ be a countable partition of the sure event: a finite or denumerably infinite set of *states*.

Let $\chi = \{X_i : \Omega \rightarrow \Re;\ i = 1, \ldots\}$ be a countable class of (bounded) real-valued random variables defined on Ω.

That is, $X_i(\omega_j) = r_{ij}$ and for each $X \in \chi$, $-\infty < inf_\Omega X(\omega) \leq sup_\Omega X(\omega) < \infty$.

Consider random variables as acts, with their associated outcomes.

	ω_1	ω_2	ω_3	$\ldots$	ω_n	$\ldots$
X_1	r_{11}	r_{12}	r_{13}	$\ldots$	r_{1n}	$\ldots$
X_2	r_{21}	r_{22}	r_{23}	$\ldots$	r_{2n}	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
X_i	r_{i1}	r_{i2}	r_{i3}	$\ldots$	r_{in}	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

***Coherence*$_1$: de Finetti's (1937) the 0-sum *Prevision Game* – wagering.**

The players in the *Prevision Game*:

- **The *Bookie* – who, for each random variable X in χ announces a *prevision* (a *fair price*), $P(X)$, for buying/selling units of X.**

- **The *Gambler* – who may make finitely many (non-trivial) contracts with the *Bookie* at the *Bookie*'s announced prices.**

For an individual contract, the *Gambler* fixes a real number α_X, which determines the contract on X, as follows.

In state ω, the contract has an *outcome* to the *Bookie* (and opposite outcome to the *Gambler*) of $\alpha_X[X(\omega) - P(X)] = O_\omega(X, P(X), \alpha_X)$.

When $\alpha_X > 0$, the *Bookie* buys α_X-many units of X from the *Gambler*.

When $\alpha_X < 0$, the *Bookie* sells α_X-many units of X to the *Gambler*.

The *Gambler* may choose finitely many non-zero ($\alpha_X \neq 0$) contracts.

The *Bookie*'s net *outcome* in state ω is the sum of the payoffs from the finitely many non-zero contracts: $\sum_{X\in\chi} O_\omega(X, P(X), \alpha_X) = O(\omega)$.

***Coherence*$_1$: The *Bookie*'s previsions {$P(X)$: $X \in \chi$} are *coherent*$_1$**

provided that there is no strategy for the *Gambler* that results in a sure

(uniform) net loss for the *Bookie*.

$$\neg\exists(\{\alpha_{X1}, \ldots, \alpha_{Xk}\}, \varepsilon > 0), \forall\omega\in\Omega \sum_{X\in\chi} O_\omega(X, P(X), \alpha_X) \leq -\varepsilon.$$

Otherwise, the *Bookie*'s previsions are *incoherent*$_1$.

The net outcome *O* is just another random variable.

The *Bookie*'s *coherent*$_1$ previsions do not allow the *Gambler* contracts where the *Bookie*'s net-payoff is uniformly dominated by *Abstaining*.

	ω_1	ω_2	ω_3	…	ω_n	…
O	$O(\omega_1)$	$O(\omega_2)$	$O(\omega_3)$	…	$O(\omega_n)$	…
Abstain	**0**	**0**	**0**	…	**0**	…

$Coherence_2$: de Finetti's (1974) *Forecasting Game* (with Brier Score)

There is only the one player in the *Forecasting Game*, the *Forecaster*.

- **The *Forecaster* – who, for random variable X in χ announces a real-valued *forecast* $F(X)$, subject to a squared-error loss outcome.**

In state ω, the *Forecaster* is penalized $-[X(\omega) - F(X)]^2 = O_\omega(X, F(X))$.

The *Forecaster*'s net score in state ω from forecasting finitely variables $\{F(X_i): i = 1, \ldots, k\}$ is the sum of the k-many individual losses

$$\sum_{i=1}^{k} O_\omega(X, F(X_i)) = \sum_{i=1}^{k} -[X_i(\omega) - F(X_i)]^2 = O(\omega).$$

***Coherence$_2$*: The *Forecaster*'s forecasts $\{F(X): X \in \chi\}$ are *coherent$_2$* provided that there is no finite set of variables, $\{X_1, \ldots, X_k\}$ and set of rival forecasts $\{F'(X_1), \ldots, F'(X_k)\}$ that yields a uniform smaller net loss for the *Forecaster* in each state.**

$$\neg \exists (\{F'(X_1), \ldots, F'(X_k)\}, \varepsilon > 0), \forall \omega \in \Omega$$

$$\sum_{i=1}^{k} -[X_i(\omega) - F(X_i)]^2 \leq \sum_{i=1}^{k} -[X_i(\omega) - F'(X_i)]^2 - \varepsilon.$$

Otherwise, the *Forecaster*'s forecasts are *incoherent$_1$*.

The *Forecaster*'s *coherent$_2$* previsions do not allow rival forecasts that uniformly dominate in Brier Score (i.e., squared-error).

	ω_1	ω_2	ω_3	…	ω_n	…
O	$O(\omega_1)$	$O(\omega_2)$	$O(\omega_3)$	…	$O(\omega_n)$	…
O'	$O'(\omega_1)$	$O'(\omega_1)$	$O'(\omega_1)$	…	$O'(\omega_1)$	…

Theorem **(de Finetti, 1974):**

A set of previsions {$P(X)$} is *coherent*$_1$.

if and only if

The same *forecasts* {$F(X)$: $F(X) = P(X)$} are coherent$_2$.

if and only if

There exists a (finitely additive) probability P such that these quantities are the P-Expected values of the corresponding variables

$$E_P[X] = F(X) = P(X).$$

Corollary**: When the variables are 0-1 indicator functions for events, *A*,**

$$I_A(\omega) = 1 \text{ if } \omega \in A \text{ and } I_A(\omega) = 0 \text{ if } \omega \notin A,$$

then de Finetti's theorem asserts:

Coherent prices/forecasts must agree with the values of a (finitely additive) probability distribution over these same events.

Otherwise, they are incoherent.

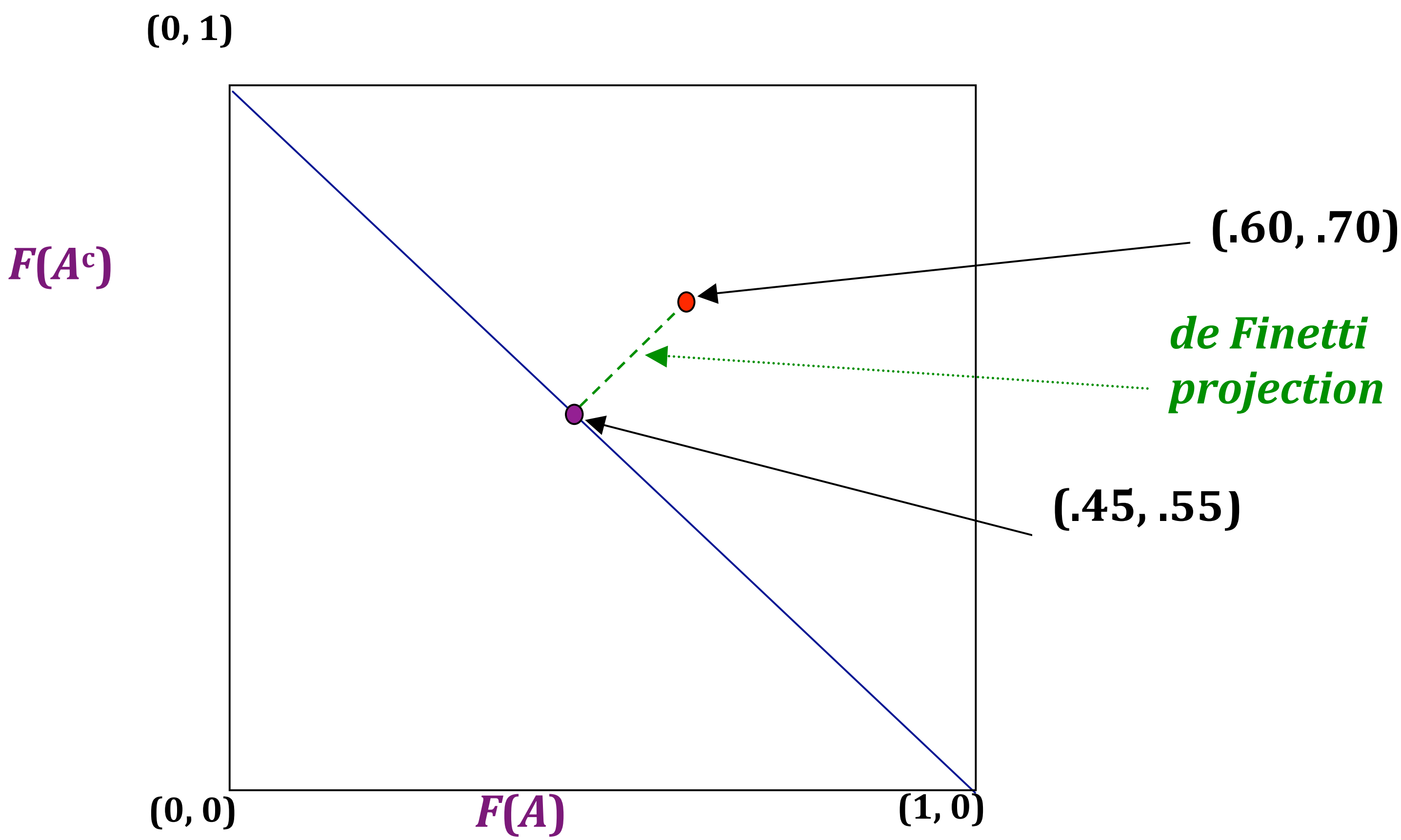
***Example*:**

A *Bookie*'s two previsions, {$P(A)=.6$; $P(A^c)=.7$}, are incoherent$_1$
The *Bookie* has overpriced the two variables.

A *Book* is achieved against these previsions with the *Gambler*'s strategy $\alpha_A = \alpha_{A^c} = 1$, requiring the *Bookie* to buy each variable at the announced price.

The net payoff to the *Bookie* is -0.3 regardless which state ω obtains.

In order to see that these are also *incoherent*$_2$ forecasts, review the following diagram


(0, 1)
F(A^c)
(.60, .70)
de Finetti
projection
(.45, .55)
(0, 0)
F(A)
(1, 0)

If the forecast previsions are not coherent$_1$, they lie outside the probability simplex. Project these incoherent$_1$ forecasts into the simplex. As in the *Example*, (.60, .70) projects onto the coherent$_1$ previsions depicted by the point (.45, .55). By elementary properties of Euclidean projection, the resulting coherent$_1$ forecasts are closer to each endpoint of the simplex. Thus, the projected forecasts have a dominating Brier score regardless which state obtains. This establishes that the initial forecasts are incoherent$_2$. Since no coherent$_1$ forecast set can be so dominated, we have coherence$_1$ of the previsions if and only coherence$_2$ of the corresponding forecasts.

Background on Coherence and Elicitation

De Finetti's interest in *coherence*$_2$, avoiding dominated forecasts under squared-error loss (Brier Score), was prompted by an observation due to Brier (1950).

Theorem **(Brier, 1950) A SEU forecaster whose forecasts are scored by squared error loss in utility units, (uniquely) maximizes expected utility by announcing her/his expected value for each forecast variable.**

- **Brier Score is a (*strictly*) *proper scoring rule*.**

That is, squared error loss provides the incentives for an SEU forecaster to be entirely straightforward with her/his forecasts.

A moment's reflection establishes that wagering, as in the *Prevision Game*, does *not* ensure the right incentives are present for the *Bookie* always to announce her/his expected $E_P(X)$ value as the "fair price" $P(X)$ for variable X.

Suppose that the *Bookie* has an opinion about the *Gambler*'s fair betting odds on an event, A.

Suppose the Bookie believes: $E_P[I_A] < E_P[I_A]$.

Then it is strategic for the Bookie to announce a prevision:

$$E_P[I_A] < P(A) < E_P[I_A].$$

The 1st contrast between two senses of coherence: *infinitely many previsions/forecasts at once.*

(1) Recall that de Finetti's coherence criteria require that the *Bookie*/*Forecaster* respects dominance only with respect to random variables created by *finite* combinations of fair-gambles/forecasts.

(2) Also, for infinite Ω, de Finetti restricted the dominance principle to require that the dominating option has *uniformly better* outcomes:

better in each state $\omega \in \Omega$ by at least some fixed amount, $\varepsilon > 0$.

Why these twin restrictions on the simple dominance principle?

The answer is because de Finetti (like, e.g., Savage) made room under a Big Tent of coherent preferences for finitely (but not necessarily countably) additive probabilities.

***Example* 1 (de Finetti, 1949).**

Let $\Omega = \{\omega_1, \ldots, \omega_n, \ldots\}$ be a denumerably infinite partition of "equally probable" states. *Bookie*'s previsions are $P(\{\omega_i\}) = 0$, $i = 1, \ldots$.

The *Bookie* judges *fair* each gamble of the form $\alpha_i(I_{\omega_i} - 0)$.
Thus, *Bookie*'s personal probability is strongly finitely additive, as

$$0 = \sum_i P(\{\omega_i\}) < P(\cup_i \{\omega_i\}) = P(\Omega) = 1.$$

These are coherent$_1$ previsions, by de Finetti's *Theorem*.

However, if the *Gambler* is allowed to engage in more than finitely many contracts at a time, even assuring that the net-outcome is finite and bounded in every state, there is a simple strategy that causes the *Bookie* to suffer a uniform (sure) loss.

$$\text{Set } \alpha_i = -1. \text{ Then, } \forall\omega\in\Omega,\ \Sigma_i\, \alpha_i(I_{\omega_i}(\omega) - 0) = -\Sigma_i\, I_{\omega_i}(\omega) = -1.$$

De Finetti noted: a sure-loss obtains in this fashion if and only if the *Bookie*'s previsions are not countably additive.

However, no such failure of dominance results by combining infinitely many forecasts, provided that the *Forecaster*'s expected score is finite.

Assume that expectations for sums of the random variables to be forecast, and also for their squares, are *absolutely convergent*:

$$\mathbf{E_P[\textstyle\sum_i |X_i|\,] \leq V < \infty} \qquad \mathbf{(1)}$$

$$\mathbf{E_P[\textstyle\sum_i X_i^2] \leq W < \infty.} \qquad \mathbf{(2)}$$

***Proposition* 1: Let $\chi = \{X_i, i = 1\ldots\}$ be a class of variables and P a finitely additive probability satisfying conditions (1) and (2), with coherent$_2$ forecasts $E_P[X_i] = p_i$.**

There does not exist a set of real numbers $\{q_i\}$ such that

$$\forall\omega \in \Omega,\ \textstyle\sum_i(p_i - X_i(\omega))^2 - \sum_i(q_i - X_i(\omega))^2 > 0.$$

***Corollary*: When conditions (1) and (2) obtain, the infinite sum of Brier scores applied to the infinite set of forecasts $\{p_i\}$ is a strictly proper scoring rule.**

Proposition 1 and its _Corollary_ establish that the two senses of coherence are _not_ equivalent when considering finitely additive probabilities and infinite sets of previsions/forecasts.

Assume the finiteness conditions (1) and (2).

Coherence$_1$, associated with the _Prevision Game_, depends upon the requirement that only finitely many _fair_ contracts may be combined at once while permitting finitely (but not countably) additive probabilities to be _coherent_.

Coherence$_2$, associated with the _Forecasting Game_, has no such restrictions for combining infinitely many forecasts. Moreover, Brier score retains its status as a strictly proper scoring rule even when infinitely many variables are forecast simultaneously.

- **Contrast #1 favors Coherence$_2$ over Coherence$_1$!**

The 2^{nd} contrast between two senses of coherence: *moral hazard*.

Consider the following case of simple dominance between two acts.

	ω_1	ω_2
A_1	*3*	*1*
A_2	*4*	*2*

Act A_2 simply dominates act A_1.

However, if there is *moral hazard* – act-state probabilistic dependence, then A_1 may maximize subjective (conditional) expected utility, not A_2.

For example, consider circumstances where $P(\omega_i \mid A_i) \approx 1$, for $i = 1, 2$.

Then, $$SE_{A_1}U(A_1) \approx 3 > 2 \approx SE_{A_2}U(A_2).$$

The agent strictly prefers A_1 over A_2.

- **With moral hazard, *simple dominance* is not compelling.**

However, there is a more restrictive version of dominance that is robust against the challenge of *moral hazard*.

Consider two acts A_1, A_2 defined by the their outcomes relative to Ω.

	ω_1	ω_2	ω_3	…	ω_n
A_1	o_{11}	o_{12}	o_{13}	…	o_{1n}
A_2	o_{21}	o_{22}	o_{23}	…	o_{2n}

Suppose the agent can compare the desirability of *all* pairs of different outcomes. The agent can compare outcome o_{ij} and o_{kl} for all pairs, and ranks them in some (strict) weak order $\prec$.

Say that A_2 <u>*robustly dominates*</u> A_1 with respect to Ω when,

$$\prec\text{-}max_{\Omega}\{o_{1j}\} \prec \prec\text{-}min_{\Omega}\{o_{1j}\}.$$

The $\prec$-best of all possible outcomes under A_1 is strictly $\prec$-dispreferred to the $\prec$-worst of all possible outcomes under A_2

- **It is immediate that *Robust Dominance* accords with SEU even in the presence of (arbitrary) moral hazards.**

***Proposition* 2: Each instance of incoherence$_1$, but not of incoherence$_2$, is a case of *Robust Dominance*.**

***Abstaining* is strictly preferred to *Book* regardless of moral hazard.**

But the same incoherent$_2$ forecast, though dominated in Brier score by a rival forecast, may have greater expected utility than that dominating rival forecast when there is moral hazard connecting forecasting and the states forecast.

***Example 2*: The *bookie* is asked for a pair of *fair* betting odds, one for an event R and one for its complement R^c.**

The same agent forecasts the same pair of events subject to Brier score.

The pair $P(R) = .6$ and $P(R^c) = .9$ are incoherent in both of de Finetti's senses, since $P(R) + P(R^c) = 1.5 > 1.0$.

For demonstrating incoherence$_1$, the *gambler* chooses $\alpha_R = \alpha_{R^c} = 1$, which produces a sure-loss of -0.5 for the *bookie*.

That is, $1(I_R(\omega) - .6) + 1(I_{R^c}(\omega) - .9) = -0.5 < 0$ in each state, $\omega \in \Omega$.

Hence, *Abstaining* from betting, with a constant payoff 0, *robustly dominates* the sum of these two *fair* bets in the partition by states Ω.

The *Forecaster* announces $F(R) = .60$ **and** $F(R^c) = .90$.

For demonstrating incoherence$_2$, consider the rival coherent forecasts

$$Q(R) = .35 \text{ and } Q(R^c) = .65,$$

the de Finetti projection of the point (.6, .9) into the coherent simplex.

For states $\omega \in R$,

the Brier score for the two *F*-forecasts is $(1-.6)^2 + (0-.9)^2 = .970$

the Brier score for the rival *Q*-forecasts is $(1-.35)^2 + (0-.65)^2 = .845$.

For states $\omega \notin R$,

the Brier score for the two *F*-forecasts is $(0-.6)^2 + (1-.9)^2 = .370$

the Brier score for the rival *Q*-forecasts is $(0-.35)^2 + (1-.65)^2 = .245$.

- **The Brier score for the rival *Q*-forecasts (.35, .65) *simply dominates*, but does <u>not</u> *robustly dominate* the Brier score for the *F*-forecasts (.6, .9) in the partition by states Ω.**

Consider a case of moral hazard in betting, or in forecasting, as before:

Let the moral hazards associated with betting be any which way at all!

Conditional on making the incoherent$_2$ F-forecasts (.6, .9), the agent's conditional probability for event R^c is nearly 1.

But conditional on making the rival (coherent) *Q*-forecasts (.35, .65) the agent's conditional probability for *R* is nearly 1.

Then it remains the case that given the incoherent$_1$ pair of betting odds (.6, .9), the *bookie* has a negative conditional expected utility of -0.5 when the *gambler* chooses $\alpha_R = \alpha_{R^c} = 1$, regardless the moral hazards relating betting with the events wagered.

Offering those incoherent$_1$ betting odds remains strictly dispreferred to *Abstaining*, which has conditional expected utility 0 even in this case of extreme moral hazard. *Abstaining* robustly dominates a *Book*.

However, with the assumed moral hazards for forecasting:

The conditional expected loss under Brier score given the incoherent$_2$ *F*-forecast pair (.6, .9) is nearly .370.

The conditional expected loss under Brier score given the rival coherent and dominating *Q*-forecast pair (.35, .65) is nearly .845.

That is, though the rival coherent$_2$ *Q*-forecast pair (.35, .65) simply dominates the incoherent$_2$ *F*-forecast pair (.6, .9) in combined Brier score, as this is <u>not</u> a case of *robust dominance*, with moral hazard it may be the that incoherent$_2$ forecast is strictly preferred.

With these moral hazards, each rival *Q′*-forecast that simply dominates the incoherent$_2$ *F*-forecast pair (.6, .9) has <u>lower</u> conditional expected utility and is dispreferred to the incoherent$_2$ *F*-forecasts.

- **Contrast #2 favors Coherence$_1$ over Coherence$_2$!**

A 3rd contrast between two senses of coherence: *state-dependent utility*.

Assume that there are no *moral hazards*:
states are probabilistically independent of acts.

Begin with a *trivial* result about *equivalent* SEU representations.

Suppose an SEU agent's $\succ$ preferences over acts on $\Omega = \{\omega_1, \ldots, \omega_n\}$ is represented by prob/state-dependent utility pair $(P;\ U_j\!: j = 1, \ldots, n)$.

	ω_1	ω_2	ω_3	$\ldots$	ω_n
A_1	o_{11}	o_{12}	o_{13}	$\ldots$	o_{1n}
A_2	o_{21}	o_{22}	o_{23}	$\ldots$	o_{2n}

$$A_2 \succ A_1 \quad \textit{if and only if} \quad \textstyle\sum_j P(\omega_j)U_j(o_{2j}) > \sum_j P(\omega_j)U_j(o_{1j}).$$

Let Q be a probability on Ω that agrees with P on null events:

$$P(\omega) = 0 \ \textit{if and only if} \ Q(\omega) = 0.$$

Let U'_j be defined as c_jU_j, where $c_j = P(\omega_j)/Q(\omega_j)$.

***(Trivial Result) Proposition* 3:**

$$(P;\ U_j) \text{ represents } \succ \quad \textit{if and only if} \quad (Q;\ U'_j) \text{ represents } \succ.$$

***Example* 3: The de Finetti *Prevision Game* for a single event *G*.**

For simplicity, let $\Omega = \{\omega_1, \omega_2\}$ with $G = \{\omega_1\}$.

Suppose that, when betting in US dollars, \$, the *Bookie* posts fair odds $P^{\$}(G) = 0.5$, so that she/he judges as *fair* contracts of the form

$$\$\alpha(I_G - .5).$$

Suppose that, when betting in Euros, €, the same *Bookie* posts fair odds $P^{€}(G) = 5/11 = 0.\overline{45}$, so that she/he judges as *fair* contracts of the form

$$€\alpha(I_G - 5/11).$$

- **Is the *Bookie* coherent$_1$? *Answer*: YES!**

- **Why do the *Bookie*'s previsions depend upon the currency?**

***Answer*: Because the *Bookie*'s currency valuations are state-dependent!**

In state ω_1		**In state ω_2**
€1 ≡ \$1.25		**€1 ≡ \$1.50**

	ω_1	ω_2
D_1	**\$1**	**\$0**
D_2	**\$0**	**\$1**

The *Bookie* is indifferent between acts D_1 and D_2 since she/he has \$-fair-betting rates of ½ on each state.

So, then the *Bookie* is indifferent between acts E_1 and E_2

	ω_1	ω_2
E_1	**€0.80**	**€0**
E_2	**€0**	**€0.67**

which mandates €-fair betting rates of 5/11 : 6/11 on $\omega_1 : \omega_2$.

***Aside*: The *Bookie* has a fair currency exchange rate of €1 ≡ \$1.375.**

But by the *Trivial Result* – there is no way to separate fair-odds (degrees of belief) from currency (utility values) based on coherent betting odds!

One (\$*P*, U_j) pair uses a state-independent utility for Dollars and a state dependent utility for Euros.

One (€*Q*; U'_j) pair uses a state-independent utility for Euros and a state dependent utility for Dollars.

- **Fixing coherent personal probabilities in the *Prevision Game* does *not* allow a separation of beliefs from values.**

What is the situation in the Forecasting Game?

What happens to the agent's coherent$_2$ forecasts when Brier score is made operational in Dollar units, rather than in Euro units?

Does propriety of squared-error loss resolve which is the *Forecaster*'s *real* degrees of belief

The answer is that the *Trivial Result* applies to ***all*** decisions over a set of acts, including those in the *Forecasting Game*.

When scored in Dollars, the coherent$_2$ *Forecaster* will maximize expected utility by offering forecasts corresponding to the ($\$P$, U_j) pair, which uses a state-independent utility for Dollars and a state dependent utility for Euros.

When scored in Euros, the coherent$_2$ *Forecaster* will maximize expected utility by offering forecasts corresponding to the (€Q; U'_j) pair, which uses a state-independent utility for Euros and a state dependent utility for Dollars.

Neither the *Prevision Game* nor the *Forecasting Game* solves the problem posed by the *Trivial Result*, the problem of separating beliefs from values based on preferences over acts.

- Contrast #3 favors *neither* Coherence$_1$ nor Coherence$_2$. Both fail !!

Summary

In three different contrasts between de Finetti's two senses of coherence, we have these varying results:

#1: Coherence$_1$ – *Previsions* immune to Book – <u>does not</u>, but
Coherence$_2$ – *Forecasting* subject to Brier score – <u>does</u>

permit the infinite combinations of previsions/forecasts that are separately coherent when these arise from a (merely) f.a. probability.

#2: Coherence$_2$ – *Forecasting* subject to Brier score – <u>does not</u>, but
Coherence$_1$ – *Previsions* immune to Book – <u>does</u>

permit arbitrary cases of *moral hazard*.

#3: Neither Coherence$_1$ – *Previsions* immune to Book,
Nor Coherence$_2$ – undominated *Forecasts* according to Brier score,

solves the challenge posed by the *Trivial Result* for separating beliefs from values based on preferences over acts.

A few references

Brier, G.W. (1950) Verification of Forecasts Expressed in Terms of Probability. *Monthly Weather Review* 78: 1-3.

de Finetti, B. (1937) Foresight: Its Logical Laws, Its Subjective Sources. (translated by H.E.Kyburg Jr.) in Kyburg and Smokler (eds.) *Studies in Subjective Probability*. 1964 New York: John Wiley, pp. 93-158.

de Finetti, B (1949) On the Axiomatization of Probability, reprinted as Chapter 5 in *Probability, Induction, and Statistics* (1972) New York: John Wiley.

de Finetti, B (1974) *Theory of Probability*, vol. 1 New York: John Wiley.

Schervish, M.J., Seidenfeld, T., and Kadane, J.B. (2011) The Effect of Exchange Rates on Statistical Decisions. Dept. of Statistics, CMU.

Seidenfeld, T., Schervish, M.J., and Kadane, J.B. (2011) Dominating Countably Many Forecasts. Dept. of Statistics, CMU.